

# Analysis of Building Structure and Topology Based on Graph Theory

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## Summary

Individual views on a building product of people involved in the design process imply different models for planning and calculation. In order to interpret these geometrical, topological and semantical data of a building model we identify a structural component graph, a graph of room faces, a room graph and a relational object graph as aids and we explain algorithms to derive these relations. The application of the technique presented is demonstrated by the analysis and discretization of a sample model in the scope of building energy simulation.

## 1 Interpretation of product model based geometry

In order to promote the application of three-dimensional modeling techniques within the design process and to substitute the still popular draft-oriented two-dimensional modeling approach, we think that it is of major importance that simulation applications themselves do support these techniques. In the paper on hand, we make a contribution to this improvement by discussing algorithms with respect to analysis and interpretation of product model based geometry in order to prepare data as input for further preprocessing.

Concerning this linking of computer aided geometric design tools with numerical simulation techniques, one of the major problems are the different *views* on a ‘building product’ of people involved in the design process. Based on these individual views, different *models* for planning and calculation are derived, i.e. the collaboration of disciplines is complicated due to the pretended lack of a common model.

Furthermore, within the design process the application of detailed simulation techniques is often avoided, because the amount of work necessary to define a numerical model is still rather high. Depending on the simulation task, a building model, for example, has to be decomposed into a set of zones in order to perform a building energy simulation, or the building topology and the components semantics have to be analyzed according to a structural simulation.

The capability of exchanging data between construction and simulation applications is desirable to remedy the time consuming and error-prone process of obtaining and sharing these data. In our opinion, the Industry Foundation Classes (IFC) are a promising basis for a solution in order to ensure software interoperability in the building industry (IAI 2003). The IFC are an object-oriented and semantical model of all components, attributes, properties and relationships of and within a building product and contain information about the design process, the whole life-cycle up to a building's disposal.

A major problem of handling geometrical information (independently whether these informations are obtained from a building product model or directly from a three-dimensional CAD model) is the *interpretation* and utilization of these data with respect to specific tasks. In order to interpret geometrical and topological data of a building model we identify a structural component graph, a graph of room faces, a room graph and a relational object graph as aids. Following the definition of these graphs, we explain algorithms to derive these relations.

## 2 Structural component graph, graph of room faces and room graph

Starting from topological and geometrical information contained in a B-rep representation of a building model, different data structures are distinguished with respect to the kind of relations being stored within a model. Among these, radial-edge or winged-edge data structures are common representatives. For a review refer to e.g. (Bungartz et al. 1996).

We identify four graphs being necessary in order to analyze the topological structure of a building and the relations between its individual components. Object semantics, if available, may represent local situations improperly due to their inherent context dependency in the sense of the above mentioned individual views on the product. Consider, for example, wall intersections or support definitions from the point of view of an architect compared to the views of a structural and an HVAC engineer.

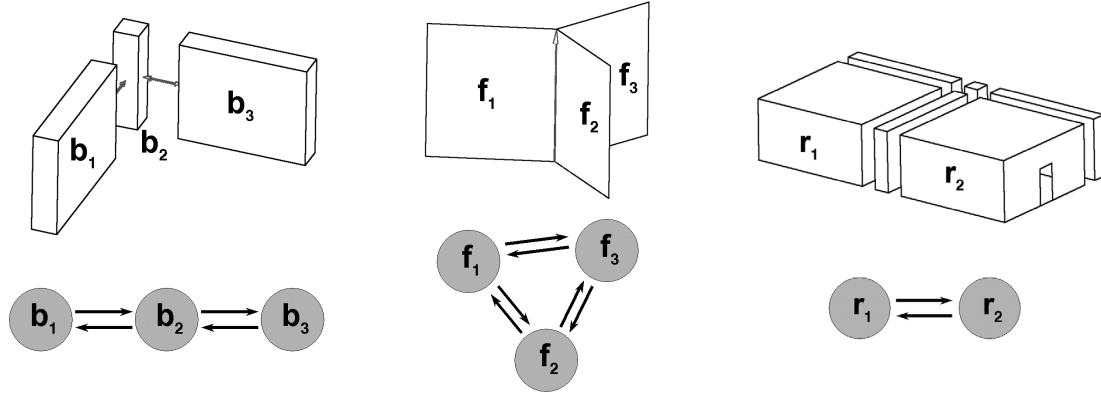


Figure 1: Structural component graph (left-hand side), graph of room faces (center), room graph (right-hand side).

Using the geometrical model of a building, we create a so-called *structural component graph*

$$G_B = (M_B; R_{PC}) \quad , \quad R_{PC} \subseteq M_B \times M_B \quad (1)$$

which defines the relation  $R_{PC}$  of plane connections between the set of all B-rep volume bodies  $M_B$ . Precondition is the decomposition of the entire model into a so-called *connection model*, which is described in the next section. Using the local regularity of a radial-edge data structure, the topological relations  $R_{NF}$  between all faces of the solid model  $M_F$  can be derived by the *graph of room faces*

$$G_F = (M_F; R_{NF}) \quad , \quad R_{NF} \subseteq M_F \times M_F \quad (2)$$

being necessary in order to extract a set of minimum closed B-rep bodies of the model, each representing an indoor air volume. Thereby, the sense of orientation is an important property of faces. Using these relations, we determine the *room-graph*

$$G_{AV} = (M_{AV}; R_R) \quad , \quad R_R \subseteq M_{AV} \times M_{AV} \quad (3)$$

by partitioning  $G_F$  into equivalence classes and subsequent condensation, which is described in Section 4. Knowing the set of indoor air volumes  $M_{AV}$ , we classify components of  $M_B$ . For example, walls can be identified as being outside, interzonal or inside walls. The latter analysis requires the definition of the relation  $R_I$ , which defines adjacencies between components  $M_B$  and air volumes  $M_{AV}$ , and can be expressed by the *relational object graph*

$$G_I = (M_B, M_{AV}; R_I) \quad , \quad R_I \subseteq M_B \times M_{AV} \quad (4)$$

The following sections describe algorithms to derive these relations.

### 3 Connection model definition

First of all, we transfer geometric data contained in a building (product) model into a solid B-rep volume model, which we denote as collectivity  $\Omega \subset \mathbb{R}^3$ . During this process, the building fabric is analyzed with respect to its layer structure. Components with intersections, such as walls, are blended accordingly. For parsing physical IFC data, we use toolbox. The procedure is described in (van Treeck 2004) in detail.

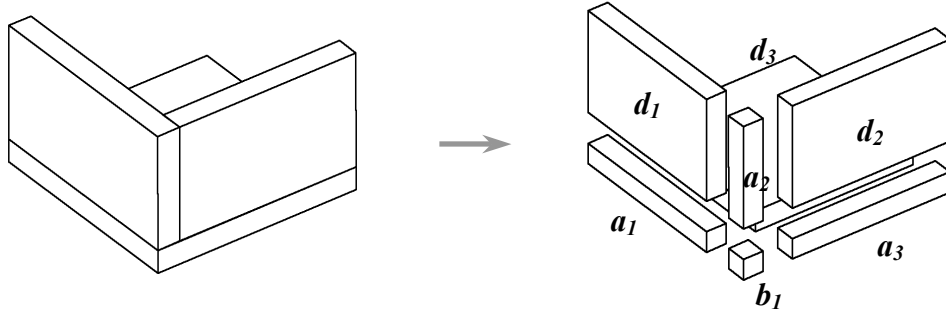


Figure 2: Example of decomposition of collectivity  $\Omega$  (left-hand side) into connection model (right-hand side).

The collectivity  $\Omega$  is decomposed into the *connection model*  $M_B$ . In order to accomplish this, we need another relational graph denoting plane connections between all components of  $\Omega$ . At those locations with coinciding elements of  $\Omega$ , components are decomposed into so-called *coupling objects*  $M_K$  and *difference objects*  $M_D$ , each representing a rigid body  $\subset M_B$ . The set of coupling objects can be further subdivided into a set  $M_{K1}$  of connection bodies of the base model and the set  $M_{K2}$  of connection bodies of the connection model (see Figure 2). We obtain

$$M_{K1} \subseteq M_K, \quad M_{K2} \subset M_K, \quad M_{K1} \cap M_{K2} = \emptyset \quad \text{and} \quad M_{K1} \cup M_{K2} = M_K \subset M_B. \quad (5)$$

Having  $r$  coupling elements  $a_i$  of the base model with  $i=1, \dots, r$ ,  $s$  coupling objects  $b_i$  of the connection model with  $j=1, \dots, s$  and  $t$  difference objects  $d_k$  with  $k=1, \dots, t$ , we can write

$$\begin{aligned} \bigcup_{i=1, \dots, r} a_i &= M_{K1} \subseteq M_K, \\ \bigcup_{j=1, \dots, s} b_j &= M_{K2} \subset M_K \quad \text{and} \\ \bigcup_{k=1, \dots, t} d_k &= M_D \subseteq M_B. \end{aligned} \quad (6)$$

Combination of both sets  $M_K$  and  $M_D$  results in the set of all connection objects  $M_B$ . Hence, the set of all points  $P_B$  of all  $n$  connection objects  $M_B$  is again part of the collectivity  $\Omega$ :

$$\begin{aligned} M_K \cup M_D &= M_B, \\ P_B &\subseteq \Omega \subset \mathbb{R}^3 \quad \text{with} \\ P_B &= \{x \in \mathbb{R}^3 : x \in v_i \wedge v_i \in M_B\} \quad \forall i = 1, \dots, n; \quad n = r + s + t \end{aligned} \quad (7)$$

The recursive decomposing process which makes use of boolean operations is described in the contribution of (Romberg 2004) within the proceedings on hand in detail. It is important to note, that, in the sense of a vef-graph based data structure, local intersections between difference objects result in common edges and/or nodes only. This behaviour will be advantageous with respect to the succeeding analysis where the connection model serves as precondition. Based on the connection model, the relation  $R_{PC}$  is obtained. It denotes the occurrence of plane connections between all coupling and difference objects. The structural component graph is an undirected and symmetric graph.

## 4 Partitioning into equivalence classes and condensation

Based on the connection model definition and the structural component graph, we now extract the set of minimum closed B-rep shells  $M_{AV}$  contained within a model, i.e. indoor air volumes and hulls. The idea is to recursively analyze adjacency relations between all faces of objects being part of the set  $M_B$ .

In order to extract air volumes, various techniques are commonly used. A well-known method makes use of the definition of the polygonal shape of a ground plane. The area being obtained is extruded using a sweeping model. A different approach utilizes an algorithm to find a convex hull based on a set of vertices, which e.g. is obtained from surrounding faces. The deficiencies of these techniques are obvious. In the former case, a unified room height is assumed and connections between rooms cannot be taken into account. The latter method is restricted to convex bodies; shapes with reentrant angles are not permitted. However, especially buildings are 'affected' by e.g. inclosed columns and typically non-convex shapes. We think that the techniques mentioned are not suitable to derive a room graph or to supply simulation tools with appropriate data (e.g. for the meshing process required for an indoor air flow simulation).

The analysis based on graph theory presented in this section allows for identifying closed and non-closed bodies contained within a model, independently whether being convex or non-convex or whether existing in a manifold or non-manifold environment. In order to reduce complexity, we shall consider plane faces only. It should be mentioned that the algorithm is universally valid, because the analysis is of intrinsically topological nature.

### 4.1 Collapsing to radial-edge data structure

The faces of all entities contained in the set of connection objects  $M_B$  are copied to the set of all faces  $M_F$ . Subsequently, the sense of orientation of these faces, i.e. the direction of their normal vectors, is reversed, in order to find indoor air volumes rather than defining rigid bodies now. Topology of a B-rep model assumes by definition that face normals point to the exterior side of a solid body.

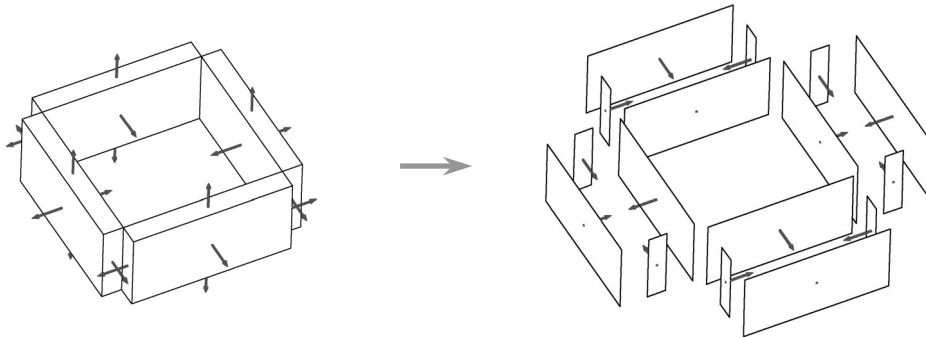


Figure 3: Inversion of the sense of orientation of all faces contained in the connection model.

Due to the dependence of the following analysis on the model consistency, we collapse the given B-rep model into a radial-edge data structure. Vertices, edges and faces are now unique, i.e. there exists exactly one instance at the same location. Vertices are conflated in an epsilon environment in order to smooth inaccuracies due to multiple model transformations and round-off errors. Inconsistencies with respect to modeling failures are detectable because the approach also provides sets of incomplete, i.e. non-closed, B-rep bodies. These modeling inaccuracies can be visualized, e.g. by highlighting the wireframe body of the boundary of a non-closed body.

The hierarchy of our radial-edge data structure implements vertices, edges, coedges, loops, faces and bodies. The basic structure was chosen in accordance with the ACIS geometric kernel (Corney 2001). Edges know about their start/end vertex and geometry and thus have an orientation. A face object aggregates a normal vector and a list of loops and 'possesses' geometry. Loops describe the polygonal shape of sub-faces by each aggregating a list of coedges. Coedges themselves serve as a topological element, point to their underlying edge and have an orientational sense with respect to this edge. Edges aggregate a set of all coedges pointing to themselves, the so-called partner coedges. Coedges are aligned in a mathematically positive sense around a corresponding face normal, if not representing a hole. Coedges of faces coinciding at common edges have a reversed sense of orientation if these faces belong to the shell of a valid closed B-rep body (see Figure 4, rule of Möbius).

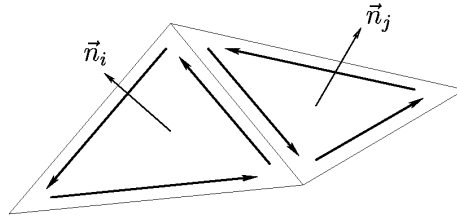


Figure 4: Rule of Möbius, explanation see text. (Bungartz 1996).

In a next step, plane connections between connection objects are removed, because these faces obviously do not belong to the value set of air volumes or hulls (see Figure 5). These connections are defined by the relation  $R_{PC}$ .

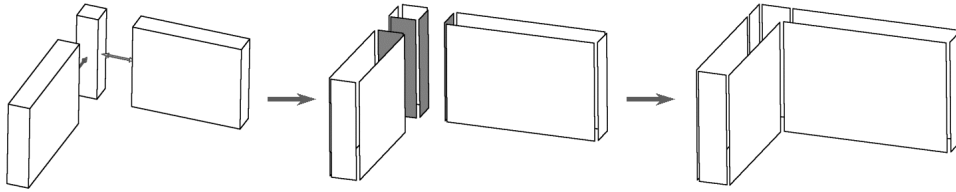


Figure 5: Removal of plane connections between components.

Furthermore, coinciding edges are detected and decomposed accordingly. For example, this case may occur in situations where objects being intermitted by an opening are in contact with a continuous floor plate (van Treeck 2004).

## 4.2 Topological analysis

Precondition for the analysis of adjacency relations between faces of the set  $M_F$  is the knowledge of the relation  $R_{NF}$  of the graph of room faces  $G_F$ . This relation depends on the *topological* and *geometrical* configuration of the elements in space. For a definition of terms it is referred to (Pahl and Damrath 2001).

We explain topological relationships using the following example. Figure 6 shows the model of an indoor air volume with inclosed column. The graph of room faces denotes connections between faces having edges in common. Nodes within the graph correspond to faces in the model. The body consists of ten faces, while two of them contain holes. Accordingly, e.g. the face at the top is adjacent to eight faces: four 'outside walls' and four faces belonging to the column. Within the graph this is equivalent to a node with eight links.

In the sense of graph theory we extract connected components of the graph  $G_F$ . Consider a relation  $R_{F,a} \subseteq M_F \times M_F$  which denotes connections between *all* faces of  $M_F$  having common edges in the sense of Figure 6. Obviously, an indoor air volume can be represented by a

connection component, while a connection component not necessarily represents an air volume. As *topological criterion* for a connection component describing a closed air volume body it is required that

- (i) the connection component must not contain leaf edges, i.e. edges being adjacent to a single face only, and
- (ii) each edge of each face has exactly one connection to another face and there exists a path between two vertices in the graph, i.e. each face in the model.

Hence, components are basic cyclic connected. Sub-graphs are blocks and free of articulation vertices.

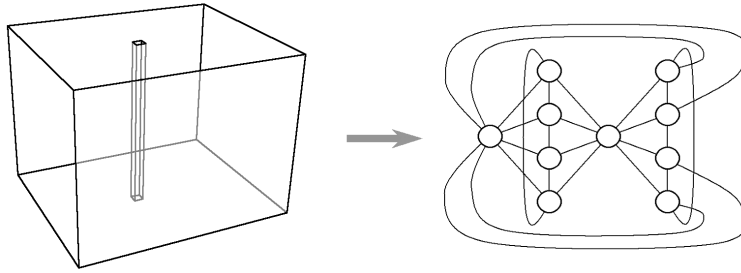


Figure 6: Graph of room faces of indoor air volume with inclosed column.

### 4.3 Geometrical analysis

We now add a *geometrical criterion* within the analysis of adjacency relations: Each connection component shall enclose a smallest volume respectively. Thereby, the validity of the sense of orientation of connected faces will play a role. Furthermore, because we intend finding a set of minimum closed B-rep bodies, we consider immediately neighboring faces with respect to their edge in common, if more than two faces are connected to that edge.

As mentioned above, the rule of *Möbius* tells us about the sense of orientation of codeges being reversed in this case. With respect to relation  $R_{F,a}$ , this implies the extension  $R_{F,o} \subseteq M_F \times M_F$  defining an appropriate correlation of the orientational sense. We write

$$R_{F,o} := \{(f_i, f_j) \in M_F \times M_F \mid \bigvee_{\vec{c}_a \in t f_i} \bigvee_{\vec{c}_b \in t f_j} (\vec{e}(\vec{c}_a) = \vec{e}(\vec{c}_b) \wedge \vec{c}_a = -\vec{c}_b)\}, \quad (8)$$

i.e. there exists a regular pair  $(f_i, f_j)$  with each element having a coedge with the same underlying edge and a reversed sense of orientation. (The subscript 't' of the operator denotes a reference to topological (sub-)elements within the hierarchy.)

If more than two faces are connected to an edge, i.e. if the set of partner coedges contains more than two elements, we have to choose the face yielding the smallest volume respectively. Thereby, the radial-edge data structure allows for organizing a set of coedges. Coedges are grouped in mathematically negative sense around their edge  $\vec{e}$ . We define a so-called radial-edge vector  $\vec{r}_i$  as cartesian product of a coedge vector  $\vec{c}_i$  and the normal vector of the underlying loop  $\vec{n}_i$ . The vector  $\vec{r}_i$  points into its corresponding face; in case of holes, the orientation is reversed.

$$\vec{r}_i := \vec{n}_i \times \vec{c}_i \quad (9)$$

Hence, the problem is reduced by one dimension. Using this radial-edge vector and according to Figure 7, angles between neighboring faces with respect to the edge  $\vec{e}$  can be readily computed and coedges can be arranged. For example, Figure 7 shows an edge  $\vec{e}$  with three connected faces. The sorted list of coedges results in the set  $\{\vec{c}_{1,3}, \vec{c}_{3,4}, \vec{c}_{2,4}\}$ .

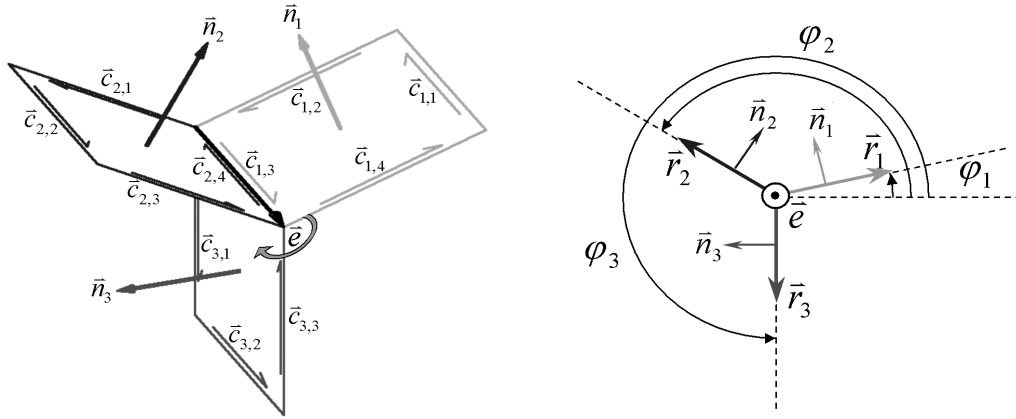


Figure 7: Analysis of spatial arrangement of coedges  $\vec{c}_i$  with respect to underlying edge  $\vec{e}$ .

The latter criterion can be expressed using the weighted relation  $R_{F,w} \subseteq M_F \times M_F$ , which evaluates the spatial arrangement of coedges with respect to an underlying edge. This relation is anti-reflexive, i.e. free of loops. The undirected graph becomes a directed graph. If we define the angle  $\varphi = \angle_{\vec{a}}(\vec{b}, \vec{c})$  with  $0 \leq \varphi \leq 2\pi$  as the angle between vectors  $\vec{b}$  and  $\vec{c}$  in a plane normal to vector  $\vec{a}$ , if vector  $\vec{b}$  is rotated in a mathematically positive sense into vector  $\vec{c}$ , we get

$$R_{F,w} := \{(f_i, f_j) \in M_F \times M_F \mid (w_{i,j} = \min)\} \quad , \quad (10)$$

where the weighting set of elements  $w_{i,j} \in W_{F,w}$  is defined by

$$W_{F,w} := \{w_{i,j} = \angle_{\vec{e}}(\vec{r}_a, \vec{r}_b) \mid \bigvee_{\vec{c}_a \in t f_i} \bigvee_{\vec{c}_b \in t f_j} (\vec{e}(\vec{c}_a) = \vec{e}(\vec{c}_b))\} \quad . \quad (11)$$

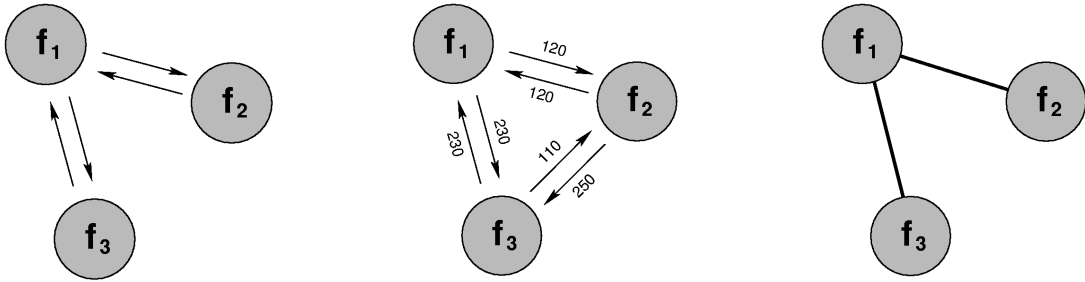


Figure 8: Visualization of relations  $R_{F,o}$  (left-hand),  $R_{F,w}$  (center) and  $R_{NF}$  (right-hand).

Consider again the example sketched in Figure 7 where the three faces  $f_1$ ,  $f_2$  and  $f_3$  are aligned around their common edge. Relation  $R_{F,o}$  describes the property of same orientations concerning adjacent faces. By joining faces  $f_2$  and  $f_3$ , we are not able to create a part of the surface of a valid volume body. Relation  $R_{F,w}$  evaluates the geometrical alignment with respect to the angular dependencies. Assume  $\varphi_1 = 30^\circ$ ,  $\varphi_2 = 150^\circ$  and  $\varphi_3 = 260^\circ$ , thus  $\delta\varphi_{1,2} = 120^\circ$  and  $\delta\varphi_{2,3} = 110^\circ$ . Neglecting the topological criterion, the next neighboring face for  $f_3$  results in  $f_2$  and not  $f_1$ , because  $110^\circ < 230^\circ$  (compare to Figure 8, center).

Concatenating relations  $R_{F,o}$  and  $R_{F,w}$  finally yields the relation  $R_{NF} \subseteq M_F \times M_F$ . Using  $R_{NF}$ , we obtain a symmetric, anti-reflexive and unweighted graph of room faces  $G_F$ .

$$R_{NF} := R_{F,o} \circ R_{F,w} \subseteq M_F \times M_F \quad (12)$$

#### 4.4 Condensation to room graph

Finally, the graph  $G_F$  can be partitioned into  $n$  basic equivalence classes  $[\lambda_i]$  with  $i=1,\dots,n$  using the equivalence relation  $Z_{NF}$ . The latter is obtained by evaluating the reflexive-transitive hull of relation  $R_{NF} \subseteq M_F \times M_F$ . The stability index  $s$  thereby is the smallest exponent if the result of the union is not affected by additional terms  $R^m$  with  $m>s$ .

$$Z_{NF} := R_{NF}^* = I \sqcup R_{NF} \sqcup R_{NF}^2 \sqcup \dots \sqcup R_{NF}^s \subseteq M_F \times M_F \quad (13)$$

Hence, the set  $M_F$  can be condensed to the quotient set  $M_{AV}$  using the mapping  $\Phi_{AV}$ . We obtain a reduced graph  $G_{AV} = (M_{AV}; R_R)$  which we denote as *room graph*:

$$\Phi_{AV} : M_F \longrightarrow M_{AV} \quad \text{with} \quad M_{AV} = \frac{M_F}{Z_{NF}} \quad (14)$$

$$R_R = \Phi_{AV}^T R_{NF} \Phi_{AV} \quad (15)$$

Hence, a closed B-rep shell is obtained if and only if a sub-graph being determined by a representative  $\lambda_i$  denotes a basic cyclic connected component, i.e. the sub-graph contains cycle and separation edges only.

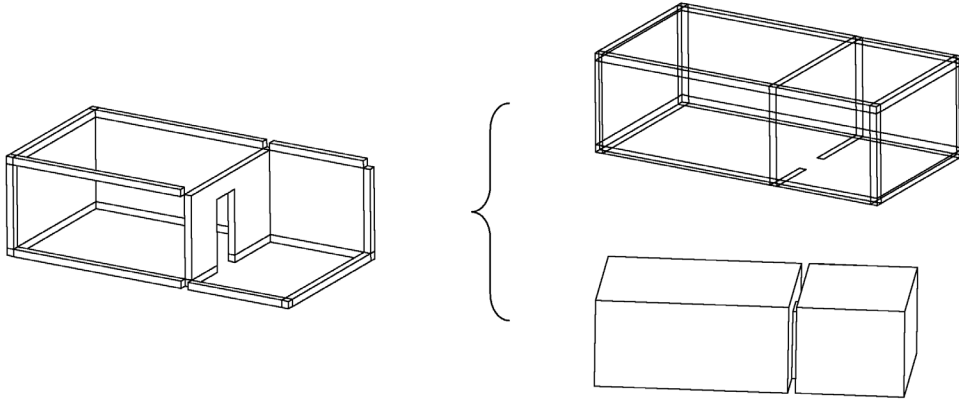


Figure 9: Indoor air volume extraction of two rooms being connected by an open door. The left-hand side shows parts of the connection model. On top of the right-hand side, an 'invalid' air volume is depicted (half space), the bottom picture shows the valid closed B-rep shell, i.e. the resulting indoor air volume.

At a first glance it may be assumed that the algorithm actually finds the set of indoor air volumes only. As an example, Figure 9 shows results being obtained in the case of two rooms being connected by an open door. Besides the air volume, we also get the hull of the model. The hull corresponds to a half space, because all face normals point to the interior.

If we use the *Gauß* integral theorem in order to compute volumes of bodies  $V([\lambda_i])$ , we can add another criterion for identifying those situations. By replacing the volume-integral in the numerical quadrature with a face-integral, we obtain negative but *finite* values as indicators.

Moreover, we separate the set  $M_{AV}$  into a set of valid indoor air volumes  $M_{AV,valid}$ , a set of invalid volumes  $M_{AV,invalid}$  and a set of non-closed B-rep bodies  $M_{AV,incomplete}$ . In cases, where the number of elements of  $M_{AV,incomplete}$  is greater than zero, we can identify modeling inaccuracies.

$$M_{AV} = M_{AV,valid} \cup M_{AV,invalid} \cup M_{AV,incomplete} \quad (16)$$

Assigning elements to their corresponding equivalence classes is achieved by forming the reflexive-transitive hull of the adjacency relation for cyclic edges  $R_{NF,Z}^*$ :



$$\begin{aligned}
[\lambda_i] \in M_{AV,valid} &\iff \lambda_i \lambda_i^T \subseteq R_{NF,\mathbf{z}}^* \wedge V([\lambda_i]) > 0 \\
[\lambda_i] \in M_{AV,invalid} &\iff \lambda_i \lambda_i^T \subseteq R_{NF,\mathbf{z}}^* \wedge V([\lambda_i]) < 0 \\
[\lambda_i] \in M_{AV,incomplete} &\iff \text{else}
\end{aligned} \tag{17}$$

In this sense, a closed B-rep shell is determined by a basic cyclic connected graph, if the above mentioned topological and geometrical criteria with respect to the definition of rigid bodies are fulfilled: Surfaces can be identified als being *closed*, faces have an *orientation* and faces are not allowed to *intersect with themselves*.

Knowledge of these graphs allows for immediately identifying building components, such as walls, being, e.g., outside or interzonal walls. We obtain the *relational object graph* by correlating structural components with air volumes. The whole procedure is described in (van Treeck 2004) in detail.

## 5 Example: Discretization within the scope of building energy simulation

As application, we briefly sketch the decomposition of a complex sample building with respect to building energy simulation. Having the room and relational object graph, it is straightforward to discretize a model for a multizone thermal building simulation or to obtain a geometric model which serves as input for mesh generation for a simulation of the interior flow regime.

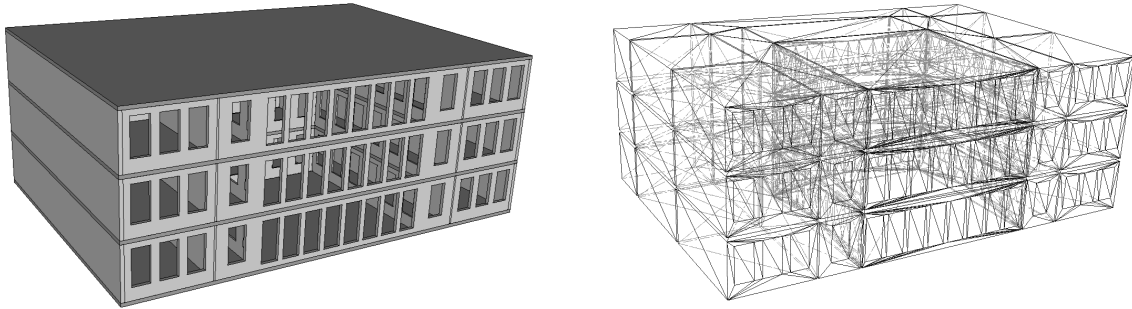


Figure 10: Sample model with atrium and triangulated surface mesh of all extracted indoor air volumes.

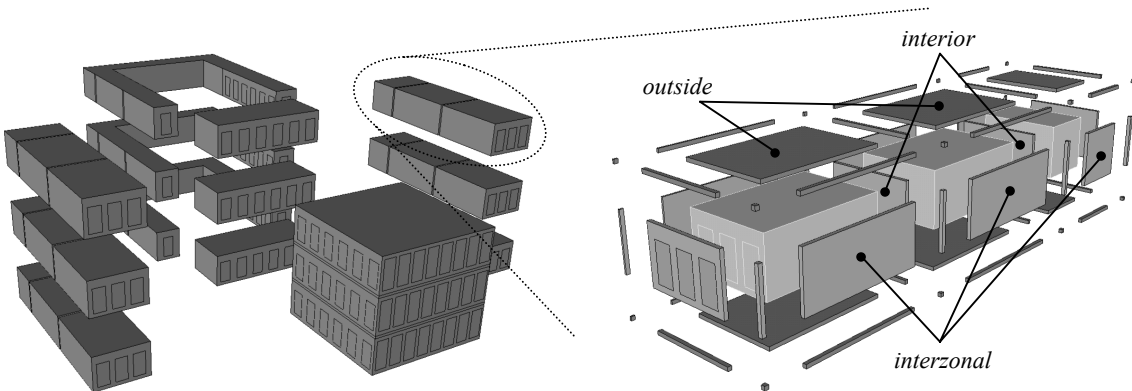


Figure 11: On the left-hand side, the set of extracted indoor air volumes is visualized. Air volumes can be grouped to zones (right-hand picture) and the semantics of building components can be identified accordingly.

Figure 10 shows on the left-hand side a sample office building consisting of three storeys with integrated atrium. Application of the presented technique yields a connection model (see right-hand detail of Figure 11), the set of all indoor air volumes contained within the model as demonstrated on the left-hand side of Figure 11 and the hull of the model. On the right-hand side of Figure 10 a (delauney-) triangulated surface mesh of all air volumes is displayed which can serve as input for a CFD code.

Using the room graph and the relational object graph it becomes obvious from Figure 11 that the semantics of building components, for example with respect to building energy simulation, can be easily obtained. Air volumes can be grouped to zones accordingly. It is important to note that this approach allows both, the derivation of an object-oriented model being required in order to set up a thermal multizone building simulator, and, on the other hand, providing geometrical data for a high-resolution simulation of building physics. Moreover, preconditions for a coupling of both techniques are accomplished. The coupling of thermal building simulation with CFD techniques is subject to a current research project.

## 6 Acknowledgements

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